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Theoretical Aspects of Genetic Algorithms

Presentation at seminar
"*Theory of Evolutionary Computation 2002*"
Max Planck Institute for Computer Science Conference Center, Schloß Dagstuhl, Saarland, Germany

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Title:

Theoretical Aspects of Genetic Algorithms. Presentation at seminar "*Theory of Evolutionary Computation 2002*", Max Planck Institute for Computer Science Conference Center, Schloß Dagstuhl, Saarland, Germany

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Key Words and Phrases:

Asymptotic convergence of genetic algorithms; unbounded power-law scaled proportional fitness selection; commuting or non-commuting crossover and mutation operators; tensor-string model; multi-set model.

Abstract:

We give an overview on results (and extensions) in Theoretical Computer Science 259 (2001), 1-61. The model considered is the tensor-string model with finite-size populations. We discuss transition of our model to the multi-set model via "permutation averaging." The analysis is based upon inhomogeneous Markov chains. Several mutation operators (single/multiple spot) are considered. Based on separate analysis of mutation M and crossover C, we discuss spectral analysis of $M\,C = C\,M$ and applications. We discuss the mutation-flow inequality $||(1-P)Mv|| \leq (1-\beta) + \beta||(1-P)v||$ where $||\cdot||$ is the Hamming norm or ℓ^1-norm, P is the projection matrix onto the space over uniform populations, and v is a probability distribution. This together with a shrinking property of fitness selection implies convergence to uniform populations for small mutation rate. Non-convergence is shown for the simple GA and other sclaed GA. Convergence to optima is shown for scaled mutation $\mu = t^{-1/L}$ and power-law scaled proportional fitness selection with logarithmic increase.

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Any Other Identifying Information of this Report:

Some of the results mentioned in the seminar/lecture at the Dagstuhl conference have been published recently in proper scientific format in Technical Report 2002-2-002 of The University of Aizu.

Distribution Statement:

First Issue: 60 copies

Supplementary Notes:

This technical report is mostly for reference purposes since in the presentation/lecture whose transparencies are reproduced several statements/theorems are listed that have not been previously appeared in the scientific literature.

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HANDWRITTEN ABSTRACT

IN

DAGSTUHL ABSTRACT BOOK

mappings"? I argue that a neutral set spans a space of different exploration distributions in the phenotype space — different genotypes may induce different topologies "around" the same phenotype (rather than being redundant). Enabling transitions between genotypes in a neutral set (by appropriate mutation operators) enables self-adaptation of the exploration distribution.

Marc Toussaint, 18 Jan. 2002

THEORETICAL ASPECTS OF GENETIC ALGORITHMS

WE GIVE AN OVERVIEW ON RESULTS (AND EXTENSIONS) IN THEORETICAL COMPUTER SCIENCE 259 (2001), 1–61. THE MODEL CONSIDERED IS THE TENSOR-STRING MODEL WITH FINITE-SIZE POPULATIONS. WE DISCUSS TRANSITION OF OUR MODEL TO THE MULTI-SET MODEL VIA "PERMUTATION-AVERAGING." THE ANALYSIS IS BASED UPON INHOMOGENEOUS MARKOV CHAINS. SEVERAL MUTATION OPERATORS (SINGLE/MULTIPLE SPOT) ARE CONSIDERED. BASED ON SEPARATE ANALYSIS OF MUTATION M AND CROSSOVER C, WE DISCUSS SPECTRAL ANALYSIS OF MC=CM AND APPLICATIONS. WE DISCUSS THE MUTATION-FLOW EQUATION

$$\| (\mathbb{1}-P_\mu) M v \|_1 \leq (1-\beta) + \beta \| (\mathbb{1}-P_\mu) v \|_1$$

$\|v\|_1=1$, WHICH TOGETHER WITH A SHRINKING PROPERTY OF FITNESS SELECTION IMPLIES CONVERGENCE TO UNIFORM POPULATIONS FOR SMALL MUTATION-RATE. NON-CONVERGENCE IS SHOWN FOR THE SIMPLE GA AND OTHER SCALED GA. CONVERGENCE (TO OPTIMA) IS SHOWN FOR SCALED MUTATION $\mu = 1/\sqrt[t]{\ }$ AND POWER-LAW SCALED PROPORTIONAL FITNESS SELECTION WITH LOGARITHMIC INCREASE.

Lothar M. Schmitt 2002/JAN/18

Performance Estimation of Some Mutation-based Evolutionary Algorithms

Anton Eremeev, Pavel Borisovsky

In this talk we discuss the upper and lower bounds on probability to generate the solutions of certain quality in the (1+1)-ES, the (1,λ)-ES evolutionary strategies and in a simplified mutation-selection genetic algorithm. The bounds are obtained in terms of the so-called monotone bounds on transition probability of the mutation operator. Some recent results on comparison of the (1+1)-ES to other mutation-based evolutionary algorithms and to the local search are presented. Also we consider some applications of the computational complexity theory to the evolutionary algorithms, showing certain limitations on capabilities of these algorithms, when they are used to solve the NP-hard problems.

Anton Eremeev, 18.01.2002

Schema Analysis

Michael D Vose,

When coarse-graining or modeling stochastic search (via schemata for example) the question of accuracy arises. Relevant to this question is the concept of compatibility of the transition function of a system with respect to an equivalence relation; this concerns the agreement between the trajectory of the system and the trajectory of the model. Compatibility is introduced by way of a simple example, and subsequently defined within the context of random heuristic search. Basic results concerning compatibility are indicated.

MD Vose 18.01.2002

TRANSPARENCIES

OF

SEMINAR / LECTURE

THEORETICAL ASPECTS OF GENETIC ALGORITHMS

LOTHAR M. SCHMITT
THE UNIVERSITY OF AIZU

LITERATURE

THEORETICAL COMPUTER SCIENCE 259 (2001), 1–61

PREVIOUS WORK WITH R.H. FUJII, C.L. NEHANIV

TENSOR-STRING MODEL

ALPHABET:

$$\mathcal{A} = \{ \tilde{a}(0), \dots \tilde{a}(\alpha - 1) \}$$

CREATURES:

$$c = (a_1, \dots a_\ell)$$

POPULATIONS:

$$p = (c_1, \dots c_s)$$

$$L = \ell \cdot s$$

USE PROBABILITY DISTRIBUTIONS IN THE FREE VECTOR SPACE OVER ALL p.

MOTIVATION FOR STRING-TENSOR MODEL

1) OVERALL SIMPLICITY

2) COMPUTER MEMORY
   POPULATION ≡ LONG BITSTRING

3) VOSE-MODEL
   CROSSOVER ≡ BILINEAR OPERATOR

$$\begin{tikzcd} & V \otimes V \arrow[d, "C_L"] \\ V \times V \arrow[ur, "\otimes"] \arrow[r, "C_B"'] & \mathcal{X} \end{tikzcd}$$

4) MAPPING TO MULTI-SET MODEL
   EASY

$$P_{\pi_s} = \frac{1}{s!} \sum_{\pi \in \Pi_s} \pi$$

MARKOV - CHAIN

$$G_t = F_t \cdot M_t \cdot C_t \quad , t \in \mathbb{N}$$

$$\prod_{\tau = t}^{1} G_\tau$$

SINGLE-SPOT MUTATION

$$M_{\mu}^{(1)} = (1 - L\mu)\, \mathbb{1} + L\mu\, M_{1/L}^{(1)}$$

$$L\mu \in (0,1)$$

$$M_{1/L}^{(1)} = \frac{1}{L} \sum_{\hat{\lambda}=1}^{L} m^{(1)}[\hat{\lambda}]$$

$$m^{(1)}[\hat{\lambda}] = \mathbb{1} \otimes \mathbb{1} \otimes \dots \underset{\uparrow \hat{\lambda}}{m^{(1)}} \otimes \dots \otimes \mathbb{1}$$

$$\mathrm{sp}(M_{\mu}^{(1)}) = 1 - \mu \frac{\alpha}{\alpha - 1} \left([0, L] \cap \mathbb{N}_0 \right)$$

R.C. GRIFFITHS / S. TAVERÉ (1997)

MULTIPLE - SPOT MUTATION

$$M_\mu^{(m)} = \prod_{\hat{\lambda}=1}^{L} \left((1-\mu)\mathbb{1} + \mu\, m^{(1)}[\hat{\lambda}] \right)$$

$$m^{(1)}[\hat{\lambda}] = \mathbb{1} \otimes \dots \otimes \mathbb{1} \otimes m^{(1)} \otimes \dots \otimes \mathbb{1}$$

$$sp\left(M_\mu^{(m)}\right) = \left\{ \left(1 - \mu \frac{\alpha}{\alpha-1}\right)^{\hat{\lambda}}, \hat{\lambda} = 0 \dots L \right\}$$

MUTATION - CROSSOVER

ASSUME $M \cdot C = C \cdot M$

e SUCH THAT $Ce = e$.

$$Me = M C^k e$$

$$= C^k M e$$

$\longrightarrow \mathrm{SPAN}_{\mathbb{C}}\{e's\}$ AS $k \to \infty$.

SPECTRAL ANALYSIS OF $M \cdot C$ IN TENSOR-STRING MODEL AND MULTI-SET MODEL.

MUTATION - FLOW EQUATION

$$\|(\mathbb{1}-P_u) M v\|_1 \leq (1-\beta)+\beta\,\|(\mathbb{1}-P_u)\,v\|_1$$

$v \in S$ (PROBABILITY DISTRIBUTION)

SINGLE SPOT $\beta = 1 - L\mu$

CROSSOVER - FITNESS EVALUATION

$$T = C \cdot F$$

$$T P_u = P_u$$

$$\| (\mathbb{1} - P_u) T v \|_1 \leq \Theta \| (\mathbb{1} - P_u) v \|_1$$

$v \in S$

1) GENETIC DRIFT

2) $\mu_\infty > \Theta$

$$\| (\mathbb{1} - P_u) v_\infty \|_1 \leq \frac{\Theta L \mu_\infty}{1 - \Theta (1 - L \mu_\infty)}$$

NON-CONVERGENCE

- SIMPLE GA
- SCALED GA

$$\mu_t \to \mu_\infty > 0$$

$$f_t = f^{g(t)} \quad , g(t) \to \infty$$

f INJECTIVE

$\Rightarrow$ LIMIT PROBABILITY DISTRIBUTION DEPENDS ONLY ON RANK

$$M_t \cdot C_t = C_t \cdot M_t$$

$\Rightarrow$ ANY C_t YIELDS SAME RESULT (LIMIT).

CONVERGENCE THEOREMS

MULTIPLE-SPOT MUTATION

$$\mu(t) = \frac{1}{L\sqrt{t}}$$

$\Rightarrow$ ERGODICITY

$$\mu(t) = \chi(t) \quad \text{CROSSOVER RATE}$$

FITNESS SCALING
(RECOMMENDED)

$$f_t = f^{g(t)}$$

$$g(t) = B \cdot \mathrm{LOG}(t+1)$$

$$\ell - 1 < L \cdot B \ \mathrm{LOG}(S_z)$$

$$\ell < S$$