Limits of the correlation function of a class of binary zero-correlation-zone sequences

Takafumi Hayashi *

June 6, 2002

Abstract

In this letter we show the estimation of the limits of correlation functions of a class of zero-correlation-zone sequences that are constructed from a set of Hadamard sequences.

1 Introduction

Binary sequences are used in various applications, In many applications involving binary sequences, the sequences are required to be uncorrelated.

For example, a code division multiple access (CDMA) system that is synchronized approximately is called an approximately synchronized CDMA (AS-CDMA). A set of sequences having a zero-correlation zone enables for an AS-CDMA system without co-channel interference [1, 2]. Such sequences can also reduce the multi-path effect of an M-ary AS-CDMA system without co-channel interference [1]. They also allow for the use of simple hardware compared with M-ary sequences.

The generation of binary sequences having a zero-correlation zone has been reported previously [3–7]. There exist two kinds of construction of of binary sequences with zero-correlation zone. The one construct the sequences from Hadamard sequences [3]. and the other construct the sequences from mutual orthogonal complementary sequences [7]. An advantage of our construction is that various kinds of sets of sequences can be constructed from Hadamard matrices derived by various methods [8–10].

Usually, a set of zero-correlation-zone sequences are used so that the phase shift of each pair of the sequences is within the zero-correlation zone. However, when we design a practical system involving a set of zero-correlation-zone sequences, we should examine the behavior of the system in the case of a phase shift go out the zero-correlation zone. We herein show the estimation of the correlation function of the zero-correlation-zone sequence derived from a set of Hadamard sequences.

*The author is with the Faculty of Computer Science and Engineering of the University of Aizu.

2 Preliminaries

In this section, we briefly introduce the notation and terminology used in this letter.

For integers a and b, let $a \oslash b$ and a % b denote the quotient and the non-negative remainder of a=b, respectively. For two sequences, $\boldsymbol{u} = [u_j]_{j=0}^{m-1}$ and $\boldsymbol{v} = [v_j]_{j=0}^{n-1}$,

$$[\boldsymbol{u}; \boldsymbol{v}] = [u_0; u_1; : : : ; u_{m-1}; v_0; v_1; : : : ; v_{n-1}]:$$

We define an operation that generates sequence of length $2l$ that consists of two sequences $\boldsymbol{v}_0 = [v_{0;j}]_{j=0}^{l-1}$ and $\boldsymbol{v}_1 = [v_{1;j}]_{j=0}^{l-1}$, both of length l. We denote this operation $\boldsymbol{c}(\boldsymbol{v}_0; \boldsymbol{v}_1)$ and it is defined as follows.

$$\boldsymbol{c}(\boldsymbol{v}_0; \boldsymbol{v}_1) = [v_{0;0}; v_{1;0}; v_{0;1}; v_{1;1}; v_{0;2}; v_{1;2}; : : : ; v_{0;l-1}; v_{1;l-1}]: \tag{1}$$

3 Sequence construction

If we have a set of n-length Hadamard sequences, we can construct a set of binary sequences. For a fixed number n, we can recursively construct a series of sets $\{\boldsymbol{s}_i^{(n;m)}\}_{i=0}^{2n-1}$ of $2n$ sequences for $m \geq 0$ as follows.

For $m = 0$, $\{\boldsymbol{s}_i^{(n;0)}\}_{i=0}^{2n-1}$ is constructed from a set of n-length Hadamard sequences, $\{\overset{n}{\boldsymbol{h}}_i\}_{i=0}^{n-1}$. Fist, a set of $2n$-length sequences, $\{\boldsymbol{g}_i^{(n)}\}_{i=0}^{2n-1}$, is constructed from the set of n-length Hadamard sequences, $\{\overset{n}{\boldsymbol{h}}_i\}_{i=0}^{n-1}$.

For $0 \leq i < n$;

$$\boldsymbol{g}_{2i}^{(n)} = \left[\overset{n}{\boldsymbol{h}}_i; \ \overset{n}{\boldsymbol{h}}_i\right]; \tag{2a}$$

$$\boldsymbol{g}_{2i+1}^{(n)} = \left[\overset{n}{\boldsymbol{h}}_i; -\overset{n}{\boldsymbol{h}}_i\right]: \tag{2b}$$

It is important to note that $\{\boldsymbol{g}_i^{(n)}\}_{i=0}^{2n-1}$ is a set of $2n$-length Hadamard sequences. It means that $\boldsymbol{g}_i^{(n)}$ are mutually orthogonal. It is also important that each $\boldsymbol{g}_{2i}^{(n)}$ is composed of even number Fourier components only and each $\boldsymbol{g}_{2i+1}^{(n)}$ is composed of odd number Fourier components only. Consequently, the cross-correlation function of $\boldsymbol{g}_{2i}^{(n)}$ and $\boldsymbol{g}_{2i+1}^{(n)}$ for any i, j have always zero value for all phase shift.

Here we denote the periodic cross-correlation function of $\boldsymbol{g}_r^{(n)}$ and $\boldsymbol{g}_s^{(n)}$ at phase shift by $\ {}_{r;\ s}^{(n)}(\ )$, where

$$ {}_{r;\ s}^{(n)}(\ ) = \sum_{j=0}^{2n-1} g_{r;j}^{(n)} g_{s;(j+\ )\%2n}^{(n)}: \tag{3}$$

This correlation function satisfies the following equations.

$$\forall r \neq s; \quad ; \qquad {}_{2r;\ s}^{(n)}(\ ) = 0; \tag{4a}$$

$$\forall r; \quad s; \quad ; \qquad {}_{2r;\ 2s+1}^{(n)}(\ ) = 0: \tag{4b}$$

Next, $\{\boldsymbol{s}_i^{(n;0)}\}_{i=0}^{2n-1}$ is constructed from $\{\boldsymbol{g}_i^{(n)}\}_{i=0}^{2n-1}$.

For $0 \leq i < n$;

$$\boldsymbol{s}_{2i}^{(n;0)} = \boldsymbol{c}\left(\boldsymbol{g}_{2i}^{(n)}; \ \ \boldsymbol{g}_{2i+1}^{(n)}\right); \tag{5a}$$

$$\boldsymbol{s}_{2i+1}^{(n;0)} = \boldsymbol{c}\left(\boldsymbol{g}_{2i}^{(n)}; -\boldsymbol{g}_{2i+1}^{(n)}\right): \tag{5b}$$

For $m > 0$, $\{\boldsymbol{s}_i^{(n;m)}\}_{i=0}^{2n-1}$ is constructed from $\{\boldsymbol{s}_i^{(n;m-1)}\}_{i=0}^{2n-1}$ as follows.

For $0 \leq i < n$;

$$\boldsymbol{s}_{2i+0}^{(n;m)} = \boldsymbol{c}\left(\boldsymbol{s}_{2i}^{(n;m-1)}; \ \ \boldsymbol{s}_{2i+1}^{(n;m-1)}\right); \tag{6a}$$

$$\boldsymbol{s}_{2i+1}^{(n;m)} = \boldsymbol{c}\left(\boldsymbol{s}_{2i}^{(n;m-1)}; -\boldsymbol{s}_{2i+1}^{(n;m-1)}\right): \tag{6b}$$

4 Properties of Synthesized Sequences

In this section, we show the properties of the sequences from our construction.

For convenience, we use Fan's notation for the zero-correlation properties of a sequence [7]. If a set of k sequences $\{\boldsymbol{v}_i\}_{i=0}^{k-1}$ of length l has a zero-correlation zone, then this set is said to be $Z(l; k; w)$, where w represents the half-width of the zero-correlation zone [7].

Theorem 1 (Zero Correlation Zone) *The set of sequences from our construction,* $\{\boldsymbol{s}_i^{(n;m)}\}_{i=0}^{2n-1}$, *is* $Z(2^{m+2}n; 2n; 2^m)$, *can be formalized as follows.*

$$\forall r \neq s; \forall \ ; | \ | \leq 2^m; \qquad {}_{r;\,s}^{(n;m)}(\ ) = 0; \tag{7a}$$

$$\forall r; \forall \ \neq 0; | \ | \leq 2^m; \qquad {}_{r;\,r}^{(n;m)}(\ ) = 0: \tag{7b}$$

Here, we denote the periodic cross-correlation function of $\boldsymbol{s}_r^{(n;m)}$ and $\boldsymbol{s}_s^{(n;m)}$ at phase shift by ${}_{r;\,s}^{(n;m)}(\ )$, where

$${}_{r;\,s}^{(n;m)}(\ ) = \sum_{j=0}^{2^{m+2}n-1} s_{r;j}^{(n;m)} s_{s;(j+\ )\%(2^{m+2}n)}^{(n;m)}: \tag{8}$$

The proof of this theorem is shown in our previous report [3]. This thorem also can be derived from the Theorem 2 in this letter.

From the definition of the $\boldsymbol{s}_i^{(n;m)}$, we can have such that

$$s_{2i;j}^{(n;m+1)} = \begin{cases} s_{i;j\oslash 2}^{(n;m)} & \text{if } j \,\%\, 2 = 0; \\ s_{i;j\oslash 2}^{(n;m)} & \text{if } j \,\%\, 2 = 1; \end{cases}$$

$$s_{2i+1;j}^{(n;m+1)} = \begin{cases} s_{i;j\oslash 2}^{(n;m)} & \text{if } j \,\%\, 2 = 0; \\ -s_{i;j\oslash 2}^{(n;m)} & \text{if } j \,\%\, 2 = 1: \end{cases}$$

Then we can also have that

$$
\begin{aligned}
{}^{(n;m+1)}_{2r;2s}(2\quad) &= {}^{(n;m+1)}_{2r;2s}(\ ) + {}^{(n;m+1)}_{2r+1;2s+1}(\ );\\
{}^{(n;m+1)}_{2r;2s}(2\quad+1) &= {}^{(n;m+1)}_{2r;2s+1}(\ ) + {}^{(n;m+1)}_{2r+1;2s}(\quad+1);\\
{}^{(n;m+1)}_{2r+1;2s+1}(2\quad) &= {}^{(n;m+1)}_{2r;2s}(\ ) + {}^{(n;m+1)}_{2r+1;2s+1}(\ );\\
{}^{(n;m+1)}_{2r+1;2s+1}(2\quad+1) &= -{}^{(n;m+1)}_{2r;2s+1}(\ ) - {}^{(n;m+1)}_{2r+1;2s}(\quad+1);\\
{}^{(n;m+1)}_{2r;2s+1}(2\quad) &= {}^{(n;m+1)}_{2r;2s}(\ ) - {}^{(n;m+1)}_{2r+1;2s+1}(\ );\\
{}^{(n;m+1)}_{2r;2s+1}(2\quad+1) &= -{}^{(n;m+1)}_{2r;2s+1}(\ ) + {}^{(n;m+1)}_{2r+1;2s}(\quad+1):
\end{aligned}
$$

These correlation function of the sequences satisfy the following equations. For even number phase shifts, we can have the following equations.

$$
{}^{(n;0)}_{2r+0;\ 2s+0}(2\ ) = {}^{(n;0)}_{2r+1;\ 2s+1}(2\ ) = {}^{(n)}_{2r+0;\ 2s+0}(\ ) + {}^{(n)}_{2r+1;\ 2s+1}(\ ); \tag{10a}
$$

$$
{}^{(n;0)}_{2r+0;\ 2s+1}(2\ ) = {}^{(n;0)}_{2r+1;\ 2s+0}(2\ ) = {}^{(n)}_{2r+0;\ 2s+0}(\ ) - {}^{(n)}_{2r+1;\ 2s+1}(\ ): \tag{10b}
$$

For odd number phase shifts, we can have the following equations.

$$
{}^{(n;0)}_{2r+0;\ 2s+0}(2\ +1) = -{}^{(n;0)}_{2r+1;\ 2s+1}(2\ +1) = {}^{(n)}_{2r+0;\ 2s+1}(\ ) + {}^{(n)}_{2r+1;\ 2s+0}(\ +1); \tag{11a}
$$

$$
{}^{(n;0)}_{2r+0;\ 2s+1}(2\ +1) = -{}^{(n;0)}_{2r+1;\ 2s+0}(2\ +1) = {}^{(n)}_{2r+0;\ 2s+1}(\ ) - {}^{(n)}_{2r+1;\ 2s+0}(\ +1): \tag{11b}
$$

We can have the following equations for even number phase shifts for $m > 0$.

$$
{}^{(n;m)}_{2r+0;\ 2s+0}(2\ ) = {}^{(n;m)}_{2r+1;\ 2s+1}(2\ ) = {}^{(n;m-1)}_{2r+0;\ 2s+0}(\ ) + {}^{(n;m-1)}_{2r+1;\ 2s+1}(\ ); \tag{12a}
$$

$$
{}^{(n;m)}_{2r+0;\ 2s+1}(2\ ) = {}^{(n;m)}_{2r+1;\ 2s+0}(2\ ) = {}^{(n;m-1)}_{2r+0;\ 2s+0}(\ ) - {}^{(n;m-1)}_{2r+1;\ 2s+1}(\ ): \tag{12b}
$$

We can have the following equations for odd number phase shifts for $m > 0$.

$$
{}^{(n;m)}_{2r+0;\ 2s+0}(2\ +1) = -{}^{(n;m)}_{2r+1;\ 2s+1}(2\ +1) = {}^{(n;m-1)}_{2r+0;\ 2s+1}(\ ) + {}^{(n;m-1)}_{2r+1;\ 2s+0}(\ +1); \tag{13a}
$$

$$
{}^{(n;m)}_{2r+0;\ 2s+1}(2\ +1) = -{}^{(n;m)}_{2r+1;\ 2s+0}(2\ +1) = {}^{(n;m-1)}_{2r+0;\ 2s+1}(\ ) - {}^{(n;m-1)}_{2r+1;\ 2s+0}(\ +1): \tag{13b}
$$

Eqs. (10)-(13) are very important for the correlation functions of the proposed sequences.

Here, we prove Eq. (10). From the definition, $s^{(n;0)}_{2i;j}$ can be denoted as $\boldsymbol{c}\left(\left[\boldsymbol{f}^{(n)}_i; \boldsymbol{g}^{(n)}_i\right]\right)$. It means that

$$
s^{(n;0)}_{2i+0;j} = \begin{cases} g^{(n)}_{2i;\ \ j\oslash 2} & \text{if } j \,\%\, 2 = 0; \\ g^{(n)}_{2i+1;j\oslash 2} & \text{if } j \,\%\, 2 = 1; \end{cases}
$$

$$
s^{(n;0)}_{2i+1;j} = \begin{cases} g^{(n)}_{2i;\ \ j\oslash 2} & \text{if } j \,\%\, 2 = 0; \\ -g^{(n)}_{2i+1;j\oslash 2} & \text{if } j \,\%\, 2 = 1: \end{cases}
$$

Therefore, we can get such that

$$
\begin{aligned}
&{}^{(n;0)}_{2r+0;\,2s+0}\left(2\ \right) = \sum_{j=0}^{4n-1} s^{(n;0)}_{2r+0;j} s^{(n;0)}_{2s+0;(j+2\ )\%4n} \\
&= \sum_{j=0}^{2n-1} s^{(n;0)}_{2r+0;2j} s^{(n;0)}_{2s+0;(2j+2\ )\%4n} + \sum_{j=0}^{2n-1} s^{(n;0)}_{2r+0;2j+1} s^{(n;0)}_{2s+0;(2j+1+2\ )\%4n} \\
&= \sum_{j=0}^{2n-1} \left(g^{(n)}_{2r+0;j} g^{(n)}_{2s+0;(j+\ )\%2n}\right) + \sum_{j=0}^{2n-1} \left(g^{(n)}_{2r+1;j} g^{(n)}_{2s+1;(j+\ )\%2n}\right): \qquad nonumber
\end{aligned}
\tag{14a}
$$

Thus have we proven Eqs. (10a). Eqs. (10b), (11), (12), and (13) can be derived similarly. From Eqs. (4) and (11) we get that

$$
{}^{(n;0)}_{2r+0;\,2s+0}\left(2\ +1\right) = -\ {}^{(n;0)}_{2r+1;\,2s+1}\left(2\ +1\right) = \ {}^{(n)}_{2r+0;\,2s+1}\left(\ \right) + \ {}^{(n)}_{2r+1;\,2s+0}\left(\ +1\right) = 0; \tag{15a}
$$

$$
{}^{(n;0)}_{2r+0;\,2s+1}\left(2\ +1\right) = -\ {}^{(n;0)}_{2r+1;\,2s+0}\left(2\ +1\right) = \ {}^{(n)}_{2r+0;\,2s+1}\left(\ \right) - \ {}^{(n)}_{2r+1;\,2s+0}\left(\ +1\right) = 0: \tag{15b}
$$

From Eqs. (12), we can get that for $m > 0$,

$$
{}^{(n;m)}_{2r+0;\,2s+1}\left(4\ \right) = \ {}^{(n;m)}_{2r+1;\,2s+0}\left(4\ \right) = 0: \tag{16}
$$

From Eq. (12), we can have such that for $m > 0$,

$$
\begin{aligned}
&{}^{(n;m)}_{2r+0;\,2s+0}\left(2^{m+1}\ \right) = \ {}^{(n;m)}_{2r+1;\,2s+1}\left(2^{m+1}\ \right) = 2\ {}^{(n;m-1)}_{2r+0;\,2s+0}\left(2^{m}\ \right) = 2\ {}^{(n;m-1)}_{2r+1;\,2s+1}\left(2^{m}\ \right) = \cdots \\
&= 2^m\ {}^{(n;0)}_{2r+0;\,2s+0}\left(\ \right) = 2^m\ {}^{(n;0)}_{2r+1;\,2s+1}\left(2\ \right):
\end{aligned}
\tag{17}
$$

From Eqs. (15) and (17), we can obtain that for $m > 0$,

$$
{}^{(n;m)}_{2r+0;\,2s+0}\left(2^m(2\ +1)\right) = \ {}^{(n;m)}_{2r+1;\,2s+1}\left(2^m(2\ +1)\right) = 2^m\ {}^{(n;0)}_{2r+0;\,2s+0}\left(2\ +1\right) = 0; \tag{18a}
$$

$$
{}^{(n;m)}_{2r+0;\,2s+1}\left(2^m(2\ +1)\right) = \ {}^{(n;m)}_{2r+1;\,2s+0}\left(2^m(2\ +1)\right) = \ {}^{(n;m-1)}_{2r+0;\,2s+0}\left(2^m(2\ +1)\right) - \ {}^{(n;m-1)}_{2r+1;\,2s+1}\left(2^m(2\ +1)\right) = 0: \tag{18b}
$$

Next, we evaluate the limit of $\left|\ {}^{(n;0)}_{2r;2s}(2\ )\right|$. Since ${}^{(n;0)}_{2r;2s}(\ ) = \ {}^{(n;)}_{2s;2r}(-\ )$, we can assume ≥ 0 without loss of generality. From Eqs. (2) and (14a), we get that

$$
\begin{aligned}
&{}^{(n;0)}_{2r;2s}(2\ ) = \sum_{j=0}^{2n-1} \left(g^{(n)}_{2r+0;j} g^{(n)}_{2s+0;(j+\ )\%2n}\right) + \sum_{j=0}^{2n-1} \left(g^{(n)}_{2r+1;j} g^{(n)}_{2s+1;(j+\ )\%2n}\right) \\
&= 2\sum_{j=0}^{n-\ -1} \left(\overset{n}{h}_{r;j} \overset{n}{h}_{s;(j+\ -n)}\right) + 2\sum_{j=n-}^{n-1} \left(\overset{n}{h}_{r;j} \overset{n}{h}_{s;(j+\ -n)}\right) + 2\sum_{j=0}^{n-\ -1} \left(\overset{n}{h}_{r;j} \overset{n}{h}_{s;(j+\ -n)}\right) - 2\sum_{j=n-}^{n-1} \left(\overset{n}{h}_{r;j} \overset{n}{h}_{s;(j+\ -n)}\right) \\
&= 4\sum_{j=0}^{n-\ -1} \left(\overset{n}{h}_{r;j} \overset{n}{h}_{s;(j+\ -n)}\right);
\end{aligned}
\tag{19a}
$$

$$
\begin{aligned}
{}^{(n;0)}_{2r;2s+1}(2\ \ ) &= \sum_{j=0}^{2n-1}\left(g^{(n)}_{2r+0;j}g^{(n)}_{2s+0;(j+\ \ )\%2n}\right) - \sum_{j=0}^{2n-1}\left(g^{(n)}_{2r+1;j}g^{(n)}_{2s+1;(j+\ \ )\%2n}\right)\\
&= 2\sum_{j=0}^{n-\ \ -1}\left(\overset{n}{h}_{r;j}\overset{n}{h}_{s;(j+\ \ -n)}\right) + 2\sum_{j=n-}^{n-1}\left(\overset{n}{h}_{r;j}\overset{n}{h}_{s;(j+\ \ -n)}\right) - 2\sum_{j=0}^{n-\ \ -1}\left(\overset{n}{h}_{r;j}\overset{n}{h}_{s;(j+\ \ -n)}\right) + 2\sum_{j=n-}^{n-1}\left(\overset{n}{h}_{r;j}\overset{n}{h}_{s;(j+\ \ -n)}\right)\\
&= 4\sum_{j=n-}^{n-1}\left(\overset{n}{h}_{r;j}\overset{n}{h}_{s;(j+\ \ -n)}\right): \qquad (19b)
\end{aligned}
$$

Then we can obtain such that

$$
\left|{}^{(n;0)}_{2r;2s}(2\ \ )\right| = 4\left|\sum_{j=0}^{n-\ \ -1}\left(\overset{n}{h}_{r;j}\overset{n}{h}_{s;(j+\ \ -n)}\right)\right| \le 4|\ \ |; \qquad (20a)
$$

$$
\left|{}^{(n;0)}_{2r;2s+1}(2\ \ )\right| = 4\left|\sum_{j=0}^{n-\ \ -1}\left(\overset{n}{h}_{r;j}\overset{n}{h}_{s;(j+\ \ )}\right)\right| \le 4(n-|\ \ |): \qquad (20b)
$$

Next, we evaluate the limit of $\left|{}^{(n;m)}_{2r;2s}(2\ \ )\right|$. We derive the following inequations by induction of m. Since ${}^{(n;m)}_{2r;2s}(\ \ ) = {}^{(n;)}_{2s;2r}(-\ \ )$, we can assume $\ \ \ge 0$ without loss of generality. From Eqs. (20), the following equatiions are satisfied for $m = 0$.

$$
\left|{}^{(n;m)}_{r;s}(2^m\ \ +\ \ ')\right| \le 4|\ \ |\left(2^m n - |\ \ '|\right): \qquad (21)
$$

From Eqs. (9), we can obtain that

$$
\begin{aligned}
\left|{}^{(n;m+1)}_{r;s}(2\ \ )\right| &\le \left|{}^{(n;m+1)}_{2r;2s}(\ \ )\right| + \left|{}^{(n;m+1)}_{2r+1;2s+1}(\ \ )\right|;\\
\left|{}^{(n;m+1)}_{2r;2s+1}(2\ \ +1)\right| &\le \left|{}^{(n;m+1)}_{2r;2s+1}(\ \ )\right| + \left|{}^{(n;m+1)}_{2r+1;2s}(\ \ +1):\right|:
\end{aligned}
$$

Then,

$$
\begin{aligned}
&\left|{}^{(n;m+1)}_{r;s}\left(2^{m+1}\ \ +2\ \ '\right)\right|\\
&\le \left|{}^{(n;m+1)}_{2r;2s}(2^m\ \ +\ \ ')\right| + \left|{}^{(n;m+1)}_{2r+1;2s+1}(2^m\ \ +\ \ ')\right|\\
&\le 2|\ \ |\left(2^m - |\ \ '|\right) + 2|\ \ |\left(2^m - |\ \ '|\right)\\
&\le 2|\ \ |\left(2^{m+1} - |2\ \ '|\right);\\
&\left|{}^{(n;m+1)}_{r;s}\left(2^{m+1}\ \ +2\ \ '+1\right)\right|\\
&\le \left|{}^{(n;m+1)}_{2r;2s+1}(2^m\ \ +\ \ ')\right| + \left|{}^{(n;m+1)}_{2r+1;2s}(2^m\ \ +\ \ '+1):\right|\\
&\le 2|\ \ |\left(2^m - |\ \ '|\right) + 2|\ \ |\left(2^m - |\ \ '+1|\right)\\
&\le 2|\ \ |\left(2^{m+1} - |2\ \ '+1|\right):
\end{aligned}
$$

Finally, we have such that

$$
\left|{}^{(n;m+1)}_{r;s}\left(2^{m+1}\ \ +\ \ '\right)\right| \le 2|\ \ |\left(2^{m+1} - |\ \ '|\right):
$$

Consequently, the following inequations are satisfied for the sequences, $\{s_i^{(n;m)}\}_{i=0}^{2n-1}$ $(m \geq 0)$.

$$\forall m \geq 0; 0 \leq r; s < n; -2n < \quad \leq 2n;$$

$$\left| {}^{(n;m)}_{2r;\;2s}\left(2^{m+1}\quad\right)\right| \leq 2^{m+2}\,|n - \quad|; \tag{22a}$$

$$\left| {}^{(n;m)}_{2r+1;\;2s+1}\left(2^{m+1}\quad\right)\right| \leq 2^{m+2}\,|n - \quad|: \tag{22b}$$

The following inequations is satisfied for the sequences of length $2^{m+2}n$, $\{s_i^{(n;m)}\}_{i=0}^{2n-1}$ $(m > 0)$. From Eq. (16), we get that

$$\forall m > 0; r; s < n; -2n < \quad \leq 2n;$$

$${}^{(n;m)}_{2r;\;2s+1}\left(2^{m+2}\quad\right) = 0; \tag{23a}$$

$${}^{(n;m)}_{2r+1;\;2s}\left(2^{m+2}\quad\right) = 0: \tag{23b}$$

Next, we show an inequation that demonstrate the limits of the correlation function of the sequences in detail.

Theorem 2 *The following equation is satisfied for the sequences of length* $4n$, $\{s_i^{(n;0)}\}_{i=0}^{2n-1}$ *(*$m > 0$*).*

$$\forall m > 0;\ 0 \leq r; s < 2n;\ -2n < \quad \leq 2n;\ -2^m < \ ' < 2^m;$$

$$\left| {}^{(n;m)}_{2r+1;\;2s}\left(2^{m+1}\ +\ '\right)\right| \leq 2\,|\ |\left(2^m n - \left|\ '\right|\right): \tag{24}$$

From this theorem, for $\ = 0$, we can obtain that

$$\forall m > 0;\ 0 \leq r; s < 2n;\ -2^m < \ ' < 2^m;$$

$${}^{(n;m)}_{2r+1;\;2s}(\ ') = 0: \tag{25}$$

5 Conclusion remarks

We evaluated the limits of the correlation function of a class of binary zero-correlation-zone sequences.

It is useful to investigate the limits of the correation functiuon of binary zero-correlation-zone sequences. This investigation will contribute to the development of new applications and will aid the understanding of the theoretical aspects of the sequences.

Acknowledgments

This study was supported in part by Grants-in-Aid for the Encouragement of Young Scientists, No. 09750083 and No. 11750060, from the Ministry of Education, Culture, Sports, Science, and Technology, Japan.

References

[1] Naoki Suehiro, "A Signal Design without Co-Channel Interference for Approximately Synchronized CDMA Systems," *IEEE Journal on Selected Areas in Communications*, vol. 12, no. 5, pp. 837–841, 1994.

[2] Takafumi Hayashi, "Novel CDMA systems by using a class of binary sequences with orthogonal subsequences and zero-correlation zone," *Technical Report of IEICE*, vol. 42, no. SST2001-42, pp. 7–14, Oct. 2001.

[3] Takafumi Hayashi, "Binary sequences with orthogonal subsequences and zero correlation zone," *IEICE Trans. Fundamentals*, vol. E85-A, no. 6, pp. 1420–1425, Jun. 2002.

[4] Takafumi Hayashi, "Uncorrelated binary sequence synthesis," *Technical Report of IEICE*, vol. 100, no. DSP-17, pp. 71–76, May 2000.

[5] Xin Min Deng and Ping Zhi Fan, "Spread Sequence Sets with Zero Correlation Zone," *Electro. Letters*, vol. 36, no. 11, pp. 993–994, May 2000.

[6] Shinya Matsufuji, Naoki Suehiro, Ping Zhi Fan, and Kenji Takatsukasa, "A Binary Sequence Pair with Zero Correlation Zone Derived from Complementary Pairs," in *ISCTA'99*. 1999, pp. 223–224, IEEE.

[7] Ping Zhi Fan and Li Hao, "Generalized Orthogonal Sequences and Their Applications in Synchronous CDMA System," *IEICE Trans. Fundamentals*, vol. E83-A, pp. 2054–2069, Nov. 2000.

[8] R. E. A. C. Paley, "On Orthogonal Matrices," *J. Math. Phys.*, pp. 311–320, 1933.

[9] Jennifer Seberry Wallis, "On the existence of Hadamard matrices," *Journal of combinatorial theory (A)*, vol. 21, no. 2, pp. 188–195, 1976.

[10] Edeard Spence, "Hadamard matrices from relative difference sets," *Journal of combinatorial theory (A)*, vol. 19, no. 3, pp. 287–300, 1975.